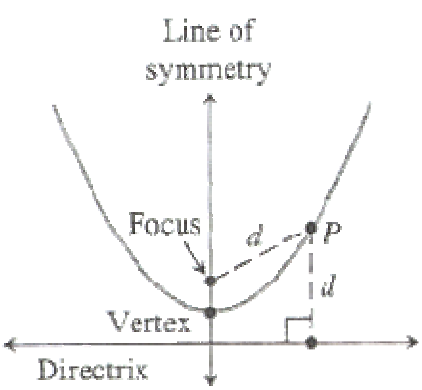
Chapter 8: Analytic Geometry in Two and Three Dimensions

Definitions

Parabola	A set of points on a plane equidistant from a line (directrix) and a point (focus)
Ellipse	Set of points on a plane whose distance from the **sum** of 2 points (foci) are equal
Eccentricity of an Ellipse	$e = \frac{c}{d} = \frac{\sqrt{a^2 - b^2}}{a}$ a=semi-major axis, b=semi-minor axis
Hyperbola	Set of points on a plane whose distance from the **difference** of 2 points (foci) are equal
Eccentricity of a Hyperbola	$e = \frac{c}{d} = \frac{\sqrt{a^2 + b^2}}{a}$ a=semi-major axis, b=semi-minor axis
Conic Section	Set of all points in a plane whose distances from a point (focus) and a line (directrix) have a constant ratio. Parabola, Ellipse, Hyperbolas, etc.

Parabolas

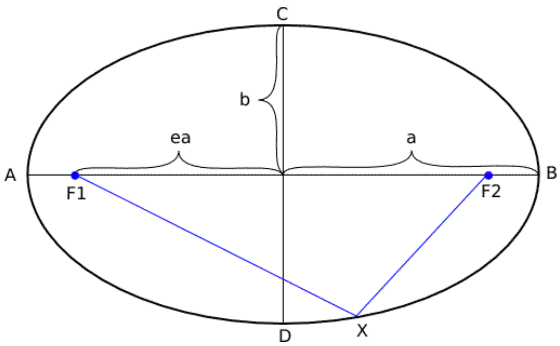
Standard Equation	$(x-h)^2 = 4p(y-k)$	$(y-h)^2 = 4p(x-k)$
Opens	Upward or downward	Left or right
Focus	(h, k + p)	(h + p, k)
Directrix	y = k – p	x = h – p
Axis	x = h	y = k
Focal length	p	p
Focal width	$\lvert 4p \rvert$	$\lvert 4p \rvert$



Note: [Focus to P] and [P to perpendicular of Directrix] are equal.

Ellipses

Standard Equation	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-h)^2}{a^2}+\frac{(x-k)^2}{b^2}=1$
Focal axis	y = k	x = h
Foci	$(h+c,k)\text{and}(h-c,k)$	$(h,k+c)\text{and}(h,k-c)$
Vertices	$(h+a,k)\text{and}(h-a,k)$	$(h,k+a)\text{and}(h,k-a)$
Semi-major	a	a
Semi-minor	b	b

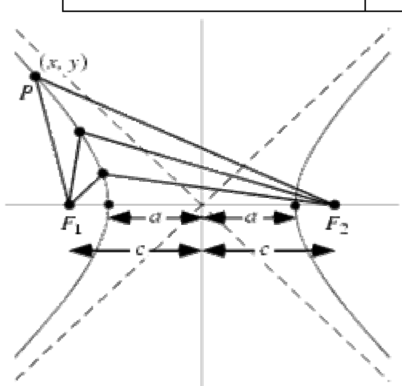


Note: The sum of [X, F1] and [X, F2] will be the same as you trace the ellipse.

- Eccentricity of an Ellipse
    - $e=\frac{c}{d}=\frac{\sqrt{a^2-b^2}}{a}$
    - E is always between 0 and 1
    - When e=0, it's a circle.

Hyperbola

Standard Equation	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-h)^2}{a^2}-\frac{(x-k)^2}{b^2}=1$
Focal axis	y=k	x=h
Foci	$(h+c,k)\text{and}(h-c,k)$	$(h,k+c)\text{and}(h,k-c)$
Vertices	$(h+a,k)\text{and}(h-a,k)$	$(h,k+a)\text{and}(h,k-a)$
Semi-transverse axis	a	a
Semi-conjugate axis	b	b
Asymptotes	$y=\frac{b}{a}(x-h)+k$ $y=\frac{-b}{a}(x-h)+k$	$y=\frac{a}{b}(x-h)+k$ $y=\frac{-a}{b}(x-h)+k$



Note: All the points of [F2, P] minus [F1, P] are equal

Axis Rotation

Standard Equation

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Discriminant Test

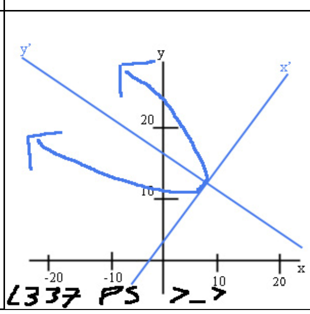
$B^2 - 4AC > 0$ then it's a Hyperbola

$B^2 - 4AC = 0$ then it's a Parabola

$B^2 - 4AC < 0$ then it's a Ellipse

How to "Un-rotate" Rotated Equation

Sample: $x^2 + 4xy + 4y^2 - 30x - 90y + 450 = 0$

1. $\cot(2\alpha) = \dfrac{A - C}{B}$	$\dfrac{1-4}{4} = \dfrac{-3}{4}$
2. Find cos(2a)	$\cos(2\alpha) = \dfrac{-3}{5}$ (Pythagorean theorem)
3. Find cos(a) and sin(a) through trig functions	$\cos(\alpha) = \sqrt{\dfrac{1 + \cos(2\alpha)}{2}} = \sqrt{\dfrac{\left[1 + \left(\dfrac{-3}{5}\right)\right]}{2}} = \dfrac{1}{\sqrt{5}}$ $\sin(\alpha) = \sqrt{\dfrac{1 - \cos(2\alpha)}{2}} = \sqrt{\dfrac{\left(1 - \dfrac{-3}{5}\right)}{2}} = \dfrac{2}{\sqrt{5}}$
4. A set of second-degree equations to plug sin(a) and cos(a) into. $A = A\cos^2\alpha + B\cos\alpha\sin\alpha + C\sin^2\alpha$ $B = B\cos(2\alpha) + (C - A)\sin(2\alpha)$ $C = C\cos^2\alpha - B\cos\alpha\sin\alpha + A\sin^2\alpha$ $D = D\cos\alpha + E\sin\alpha$ $E = E\cos\alpha - D\sin\alpha$ $F = F$	$A = 5$ $B = 0$ $C = 0$ $D = \dfrac{-42}{\sqrt{5}}$ $E = -6\sqrt{5}$ $F = 450$ $5x^2 - 42\sqrt{5}x - 6\sqrt{5}y + 450 = 0$
5. Put equation into the standard parabola, ellipse, or hyperbola based on the results of the **Descriminant Test**	Using descriminant test, the original equation was a rotated parabola. We find that $h = \dfrac{21}{\sqrt{5}}$ and $k = \dfrac{3\sqrt{5}}{10}$
6. Graph with hand or graphing calc. Realize that the graphed equation is not the original equation but Un-rotated so that it can be paralleled to the X and Y axis.	 The blue X and Y axis are the new axis on the transformed graph. The black axis' are the original graph.